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PRECISE DIFERENTIATION CAN SIGNIFICANTLY IMPROVE THE ACCURACY OF OPTICAL FLOW MEASUREMENTS

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ABSTRACT

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Keywords: Times Roman, image area, acronyms, references

1. INTRODUCTION

Measuring objects' displacements and deformation through analysis of local image motions in image sequences is required in many computer vision and optical metrology applications. Estimation of local image motion based upon local derivatives in a sequence of images is, after the paper by B. Horn and B. Schunck[1], commonly called optical flow[2]. The basic principle of optical flow computation can be described as follows. Let $I(x,y,t)$ be image intensity defined in spatial (x,y) and time t coordinates and during time interval Δt pixel (x,y) moves, due to the object motion, to point $(x+\Delta x, y+\Delta y)$. Let also assume that the object motion causes no changes in pixel intensity, and the changes may occur solely due to random factors such as additive signal independent white Gaussian noise that can be attributed to image sensor. Then, given image intensity measurements $I(x,y,t)$ and $I(x+\Delta x, y+\Delta y, t+\Delta t)$ in two time moments t and $t+\Delta t$, statistically optimal maximum likelihood estimation of movement vector $(\Delta x, \Delta y)$ is found as a solution of the equation:

$$(\Delta x, \Delta y, t) = \underset{(\Delta x,\Delta y)}{\arg\min} \iint_{(\xi,\eta)\in ARM} \left[I(\xi,\eta,t) - I(\xi+\Delta x, \eta+\Delta y, t+\Delta t) \right]^2 d\xi d\eta \qquad (1)$$

where $ARM(x,y)$ is the object Area of Rigid Motion centered at the point (x,y). Within the accuracy of Taylor expansion of the image intensity function $I(x,y,t)$, Eq. (1) can be approximated by the equation

$$(\Delta x, \Delta y, t) = \underset{(\Delta x,\Delta y)}{\arg\min} \iint_{(\xi,\eta)\in ARM} \left[\frac{\partial I(\xi,\eta,t)}{\partial \xi}\Delta x + \frac{\partial I(\xi,\eta,t)}{\partial \eta}\Delta y + \frac{\partial I(\xi,\eta,t)}{\partial t}\Delta t \right]^2 d\xi d\eta =$$

$$\underset{(\Delta x,\Delta y)}{\arg\min} \iint_{(\xi,\eta)\in ARM} \left(\dot{\mathbf{I}}_{\xi,\eta,t} \bullet \mathbf{\Delta XYT} \right)^2 d\xi d\eta \,, \qquad (3)$$

where $\mathbf{\Delta XYT}$ is a vector of space-time shifts $(\Delta x; \Delta y; \Delta t)$ and $(.\bullet.)$ is a scalar (inner) vector product, $\dot{\mathbf{I}}_{\xi,\eta,t}$ is a vector of image intensity function space-time derivatives: $\dot{\mathbf{I}}_{\xi,\eta,t} = \left(\frac{\partial}{\partial \xi} I(\xi,\eta,t); \frac{\partial}{\partial \eta} I(\xi,\eta,t); \frac{\partial}{\partial t} I(\xi,\eta,t) \right)$,

.

Two original algorithms of optical flow computation, Lucas-Kanade ([3, 4]) and Horn-Shunk ([2]) ones implement modifications of Eq. 3. In the Lucas-Kanade algorithm, a weighting window function $W(\xi,\eta)$ is introduced:

$$(\Delta x, \Delta y, t) = \underset{(\Delta x, \Delta y)}{\arg\min} \iint_{(\xi,\eta)\in ARM} W(\xi,\eta)\left(\dot{\mathbf{I}}_{\xi,\eta,t} \bullet \mathbf{\Delta XYT}\right)^2 d\xi d\eta \quad (5)$$

while in the Horn-Shunk algorithm an additional constrain on smoothness of the space shift vector is introduced and integration is extended to the entire image frame ($ImgFr$):

$$(\Delta x, \Delta y, t) = \underset{(\Delta x, \Delta y)}{\arg\min} \iint_{(\xi,\eta)\in ImgFr} \left\{\left(\dot{\mathbf{I}}_{\xi,\eta,t} \bullet \mathbf{\Delta XYT}\right)^2 + \lambda^2 \nabla^2\right\} d\xi d\eta \,, \quad (6)$$

where

$$\nabla^2 = \left(\frac{\partial \Delta_x}{\partial \xi}\right)^2 + \left(\frac{\partial \Delta_x}{\partial \eta}\right)^2 + \left(\frac{\partial \Delta_y}{\partial \xi}\right)^2 + \left(\frac{\partial \Delta_y}{\partial \eta}\right)^2 , \quad (7)$$

and λ^2 is an Euler-Lagrange multiplier.

Since Horn-Schunk's and Lucas-Kanade's works[2-4], a number of modifications of these basic optical flow algorithms have been made[5,7,8] aimed at improvement of the accuracy and reliability of optical flow computations. Most of the efforts deal with the problem of local minima in the optimization process, possible intensity variations that cannot be described by the additive normal noise model, inaccuracies due to image sampling, improvement of numerical optimization schemes.

As one can see from the above discussion, all optical flow methods rely on computing spatial and time derivatives of the image intensity function. However, to the best of the present authors' knowledge, no attempts have been made to investigate the influence of the accuracy of computing image function derivatives from sampled image data on the accuracy of optical flow estimations, though some authors did appreciate the importance of accurate computation of derivatives (see, for instance[2]). In this paper, we compare the accuracy of optical flow estimation for three methods implemented with the use of different methods of numerical differentiation and show, that the exact calculation of the derivatives can significantly improve the performance of the optical flow algorithms.

2. NUMERICAL DIFFERENTIATION ALGORITHMS

The following five numerical differentiation methods considered were standard numerical differentiation algorithms D1, D2 and D4[10] and Simoncelli Kernel[2] implemented as digital convolutions with convolution kernels

$$h^{D1} = [-1,1] \quad (8, a)$$

$$h^{D2} = [-1,0,1] \quad (8, b)$$

$$h^{D4} = [-1/12, 2/3, 0, -2/3, 1/12] \quad (8, c)$$

$$h^{Simoncelli} = [-0.108 \;\; -0.283 \;\; 0.0 \;\; 0.283 \;\; 0.108] \quad (8, d)$$

and exact numerical differentiation FFT-based algorithms implemented as filtering in DFT and an DCT domain[11]. DFT domain numerical differentiation algorithm is described by the equation:

$$\left\{\dot{a}_k^{DFT}\right\} = \mathbf{IFFT}_N\left(\left\{\eta_r^{(diff)}\right\} \bullet \mathbf{FFT}\left(\left\{a_k\right\}\right)\right) \quad (9)$$

where $\{a_k\}$ and $\dot{a}_k^{DFT}$ samples of signal and its derivative, respectively, $k = 0,...,N-1$, and

$$\eta_r^{(diff)} = \begin{cases} -i2\pi r/N, & r = 0,1,...,N/2-1 \\ -\pi/2, & r = N/2 \\ i2\pi(N-r)/N, & r = N/2+1,...,N-1 \end{cases} \qquad \eta_r^{diff} = \begin{cases} -i2\pi r/N, & r = 0,1,...,(N-1)/2-1 \\ i2\pi(N-r)/N, & r = (N+1)/2,...,N-1 \end{cases} \quad (10)$$

for even N and for odd N, correspondingly. DCT domain numerical differentiation algorithm is described by the equation

$$\{\dot{a}_k\} = \mathbf{IDcST}\left(\left\{\eta_{r,opt}^{(diff)}\right\} \bullet \mathbf{DCT}(\{a_k\})\right) = \frac{2\pi}{N\sqrt{2N}} \sum_{r=1}^{N-1} r\alpha_r^{(DCT)} \sin\left(\pi \frac{k+1/2}{N} r\right) \quad (11)$$

Frequency responses of five numerical differentiation methods are plotted in Fig. 1, in which one can see that the standard differentiation methods Simoncelli, D2 and D4 tend to produce significant errors for signal with bandwidths higher than 0.5 of the base-band defined by the signal sampling rate, while the DFT-based differentiation algorithm implements exact differentiation of sampled signals and the frequency response of the DCT-based algorithm is very close to that of the DFT based algorithm. The advantage of the DCT–based algorithm is that it is substantially less vulnerable to boundary effects, which are unavoidable in digital filtering because of the finite number of signal samples.

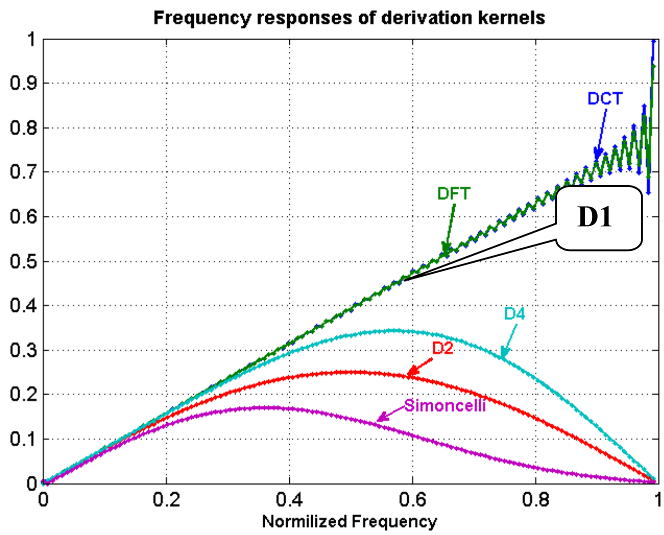


Fig. 1. Frequency responses of five numerical differentiation methods\

3. OPTICAL FLOW ERROR LOWER BOUND

Lower bound of error in estimating optical flow does not depend on the methods of solving Eq. 3 and is determined by two factors: errors in evaluation of spatial derivatives and errors due to truncation of Taylor expansion.
Give analytical formulas of StdDevError for different displacements vs signal bandwith and comparison plots (for displacements 0.1 0.2 0.4 0.6 0.8 1).

4. METHODOLOGY OF THE COMPARISON OF OPTICAL FLOW COMPUTATION METHODS

For the comparison purpose, the following three optical flow algorithms have been selected: MS-Lukas-Kanade's, MS-Horn-Schunk's and Brox's et al. ones. The first two methods are our implementations of the well-known basic techniques [2-4], which we embed into the multi-scale (multi-grid) framework in order to make them be comparable with one of the most recent algorithm by Brox et al.[5]. In what follows, we refer to these algorithms as MS-Lukas-Kanade's and MS-Horn-Schunk's ones.

A crucial issue of the comparison of optical flow algorithms is selecting test images, for which the "ground truth" of the optical flow is known. In the literature, for comparison of the accuracy of optical flow estimation, a test image "Yosemite" is commonly used for which optical flow was directly measured by Lynn Quam and is available on *ftp://ftp.csd.uwo.ca/pub/vision*. The availability of the optical flow "ground truth" is the only justification for the use of this test image, and it is not clear at all how well this image represents all possible images and scenes for which computing optical flow might be required. In addition, it was recently found, that the "ground truth" data available for the "Yosemite" image, are, in fact, not perfectly correct [8]. Therefore, because the accuracy of numerical differentiation depends of the signal bandwidth, we have chosen to use as test images, in addition to the "Yosemite" image, a set of pseudo-random images of 256x256 pixels with uniform spectrum within a circular fraction of the base band defined by the image sampling rate. The ratio of the radius of this circle to the highest spatial frequency, which in our case was equal to 128, was used as a numerical parameter defining the type of test image. Examples of such test images are shown in Fig. 2.

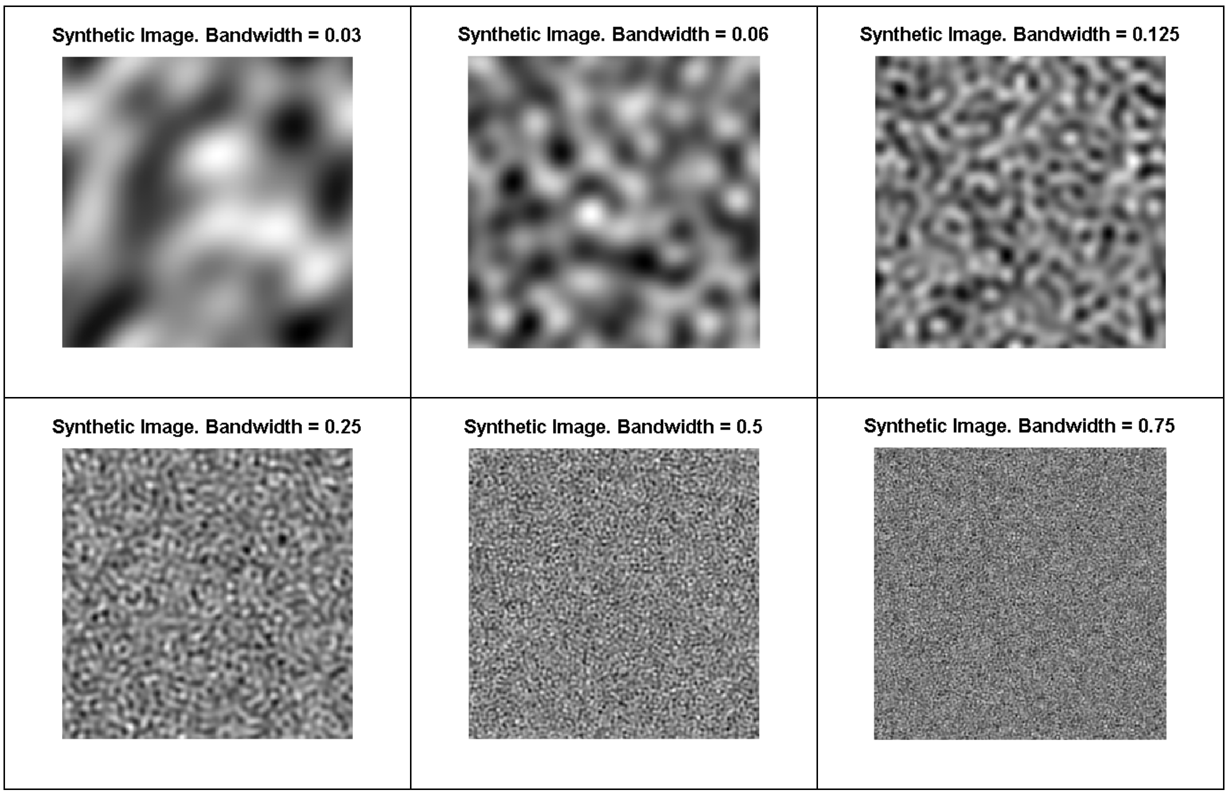


Fig. 2 Examples of test pseudo-random images with different bandwidth

In order to have a firm basis for comparison of the performance of different optical flow method, we introduced artificial shifts in the pseudo-random test images as well as in the "Yosemite" test image. First of all, image shifts by the integer number of inter-pixel distance were used, which guarantied the absence of interpolation errors in the shifted test images. We also generated images with global half inter-pixel distance shifts, using for this purpose discrete sinc-inerpolation, which is proved to be the perfect interpolation method for sampled data with finite number of samples[11].

Displacing source test images in this manner we obtain, for each specific test image, the consecutive frame with exactly known displacement vector for each pixel in the frame. These pairs of test images were used to run optical flow algorithms with above listed modifications of numerical differentiation. Note that the derivatives over the time variable were, in these experiments, found as inter-pixel differences in the consecutive frames. Therefore, the performance of only spatial differentiations in optical flow computation was studied.

In all set of experiments, global shifts were used and statistics of optical flow coordinate shift evaluation errors (distribution density histograms, standard deviation and mean values) were found over the set of all image pixels, the errors been computed as differences between pixel $\boldsymbol{x}$ and $\boldsymbol{y}$ shifts found by the algorithm and the known ones. In order to avoid boundary effects in differentiation by means of DFT based algorithm, the errors were analyzed within "safe" internal area of images separated from image borders by margins of 32 pixels.

5. EXPERIMENTAL RESULTS AND DISCUSSION

The following numerical parameters are taken optimized for the best algorithm performance over the mentioned above set of images). For **all??** algorithms there are 10 scale levels with reduction factor 0.8. The MS-Lucas-Kanade optimization window is 5x5. The MS-Horn-Schunck smoothness parameter λ is 5 and the Gauss-Seidel iterations number is 50. For Brox et. al. there are 7 inner fixed point iterations and 10 SOR interations. The smoothness parameter $\boldsymbol{\alpha = 20}$ and the gradient constancy weight $\boldsymbol{\gamma = 5}$.

Experimental results are summarized in Figs. 3 and 4 and tables 1 and 2 for pseudo-random test images and, for the "Yosemite" test image, in Fig. 5 and table 3.

Fig. 3 shows, for different differentiation methods and for three optical flow algorithms, how standard deviation of the coordinate shift error depends on the pixel shifts, introduced to the pseudo-random test images. The test image bandwidth parameter in these experiments was set to 0. 75. An example of such image is shown in Fig. 2 (lower right image). From this figure one can clearly see that the more accurate differentiation translates to the better accuracy of optical flow estimation. One can also see that DFT and DCT-based methods are practically identical in terms of the accuracy they provide. In Table 1, these data are represented in a form of the accuracy gain factor found as the ratio of estimation error standard deviation for differentiation methods D2 and D4 to that for DCT-based differentiation method.

Plots in Fig. 4 illustrate how standard deviation of the optical flow evaluation methods depends on the bandwidth parameter of the test images for different differentiation methods and the three selected optical flow algorithms. From the plots one can see that substantial gain in the optical flow evaluation accuracy can be obtained, with better differentiation techniques, for images that contain a substantial high frequency content. Same data are represented in Table 2 in form of the accuracy gain factor provided by the DCT-based differentiation.

From Fig. 5 for the "Yosemite" test image, one can clearly see that less accurate differentiation methods do tend to produce higher optical flow estimation error in image areas rich of high frequencies marked with lighter pixels in Fig. 5, b), which shows local image intensity standard deviation in the window of 3x3 pixels. The quantitative data on the standard deviation of the optical flow estimation error obtained on the "Yosemite" test image with global shift by one inter-pixel distance in both coordinates are summarized in Table 3. These data also demonstrate improvement in the accuracy of optical flow estimation achieved with more accurate differentiation techniques.

In conclusion, we have to mention that this improvement was not uniform over the images. In some areas, specifically, in those areas where the optical flow estimation error were particularly small compared to those in other areas we sometimes observed, for the Brox's algorithm, a paradoxical phenomenon: more accurate differentiation methods D4 and DCT produced optical flow estimations with larger errors then the simple D2 method. This phenomenon requires further analysis and, perhaps, can be attributed to peculiarities of the optimization procedure used in the algorithm. We have to also admit that because of unavailability of the original implementation of the algorithm, we used our own implementation built on the base of the description of the algorithm in Ref. 5. We, however, believe that this fact does not compromise the general conclusion on the substantial potential gain in the accuracy of optical flow estimation that can be achieved with more accurate differentiation methods.

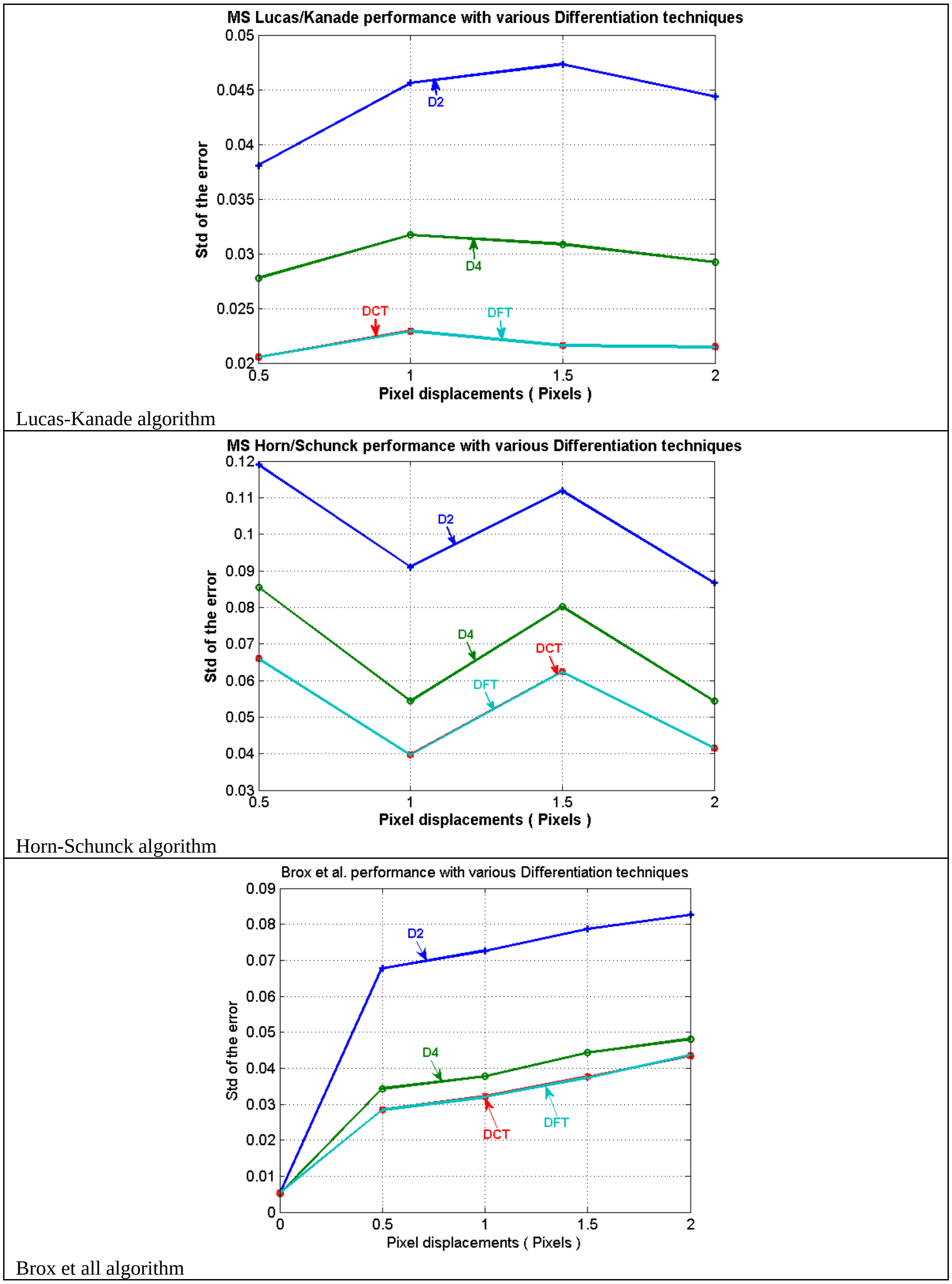
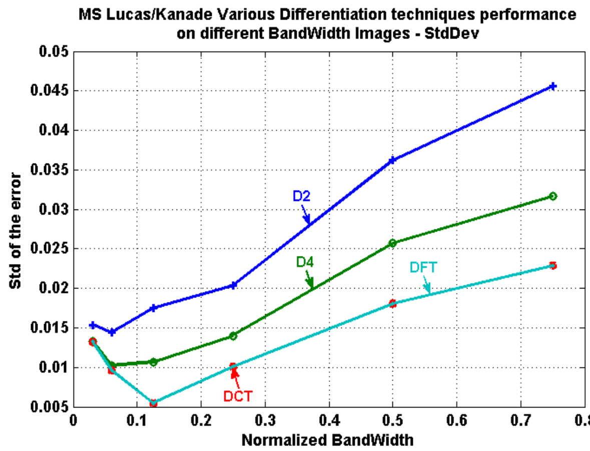
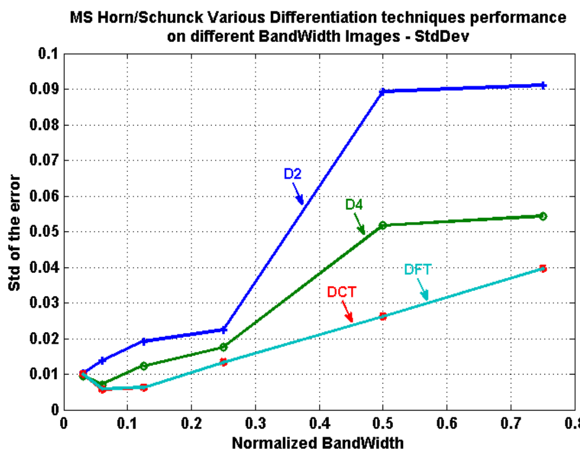


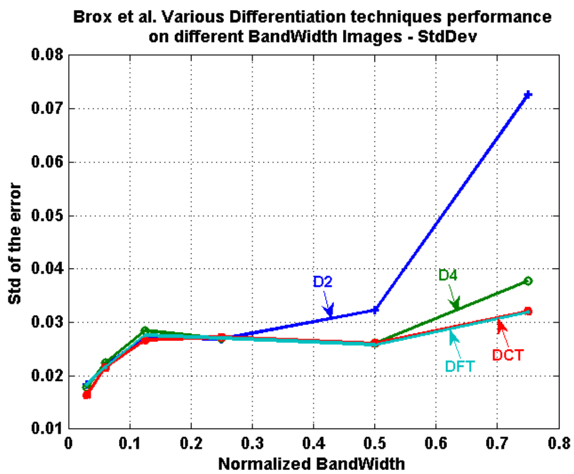
Fig. 3 Standard deviations of coordinate shift error for pseudo-random test image with bandwidth 0.75 for 3 optical flow algorithms and different differentiation methods and different displacements (in the units of inter-pixel distance)



Lucas-Kanade algorithm

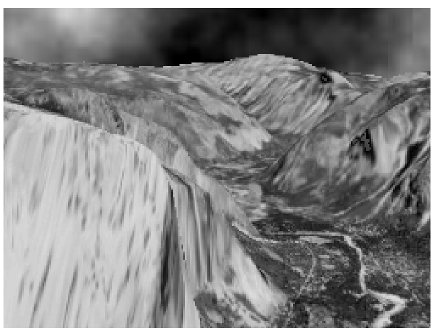


Horn-Schunk algorithm performance

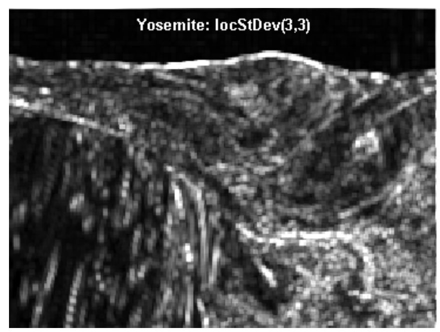


Brox et al. algorithm performance

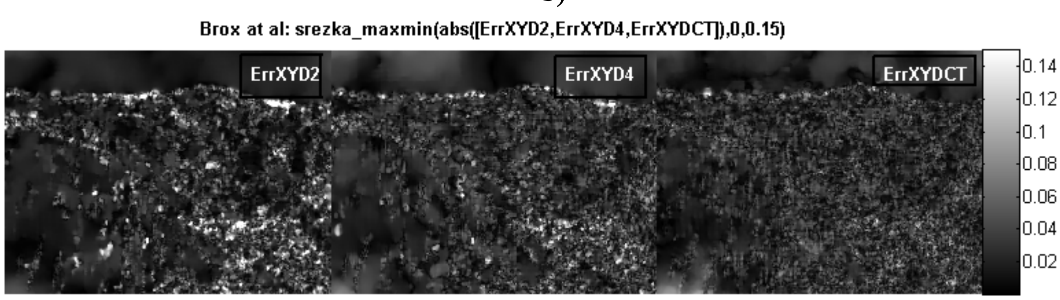
Fig. 4. Standard deviations of coordinate shift error (in the units of inter-pixel distance) for pseudo-random test images of bandwidths from 0.03 to 0.75 with one pixel displacement for 3 optical flow algorithms and different differentiation methods



a)



b)



c)

Fig. 5. Test image 'Yosemite' (a), its local standard deviation in the window 3x3 (b) and maps of absolute errors of optical flow calculation for three methods of computing derivatives: D2, D4 and DCT (c) for the global shift of one inter-pixel distance in both coordinates.

Table 1. The accuracy gain factor (ErrStDev[D2,D4])/ErrStDev[DCT])

Optical flow computation method	Numerical differentiation method	Displacement (pixels)			
		0.5	1	1.5	2
MS-Lukas-Kanade	*D2*	1.9	2.0	2.2	2.1
	D4	1.4	1.4	1.4	1.4
MS-Horn-Shunk	*D2*	1.8	2.3	1.8	2.1
	D4	1.3	1.4	1.3	1.3
Brox et all	*D2*				
	D4				

Table 2. Accuracy gain factor (Err StDev[D2,D4])/ErrStDev[DCT])

Optical flow computation method	Numerical differentiation method	Image bandwidth (in fraction of the base band)					
		0.03	0.06	0.125	0.25	0.5	0.75
MS-Lukas-Kanade	D2	1.2	1.5	3.2	2.0	2.0	2.0
	D4	1.0	1.1	2.0	1.4	1.4	1.4
MS-Horn-Shunk	D2	2.4	3.1	1.7	3.4	2.3	2.4
	D4	1.2	2.0	1.3	2.0	1.4	1.2
Brox et all	D2						
	D4						

Table 3. Standard deviation of optical flow estimation on the "Yosemite" test image for global shift by one inter-pixel distance in both coordinates

Optical flow computation method	Differentiation method		
	D2	D4	DCT
MS-Lukas-Kanade	0.073	0.055	0.05
MS-Horn-Schunk	0.23	0.21	0.12
Brox et al.			

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