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Real-Time Vision-Based Traffic Flow Measurements and Incident Detection

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ABSTRACT

Visual surveillance for traffic systems requires short processing time, low processing cost and high reliability. Under those requirements, image processing technologies offer a variety of systems and methods for Intelligence Transportation Systems (ITS) as a platform for traffic Automatic Incident Detection (AID). There exist two classes of AID methods mainly studied: one is based on inductive loops, radars, infrared sonar and microwave detectors and the other is based on video images. The first class of methods suffers from drawbacks in that they are expensive to install and maintain and they are unable to detect slow or stationary vehicles. Video sensors, on the other hand, offer a relatively low installation cost with little traffic disruption during maintenance. Furthermore, they provide wide area monitoring allowing analysis of traffic flows and turning movements, speed measurement, multiple-point vehicle counts, vehicle classification and highway state assessment, based on precise scene motion analysis.

This paper suggests the utilization of traffic models for real-time vision-based traffic analysis and automatic incident detection. First, the traffic flow variables, are introduced. Then, it is described how those variables can be measured from traffic video streams in real-time. Having the traffic variables measured, a robust automatic incident detection scheme is suggested. The results presented here, show a great potential for integration of traffic flow models into video based intelligent transportation systems. The system real time performance is achieved by utilizing multi-core technology using standard parallelization algorithms and libraries (OpenMP, IPP).

Keywords: Real-Time Traffic Measurements, Intelligent Transportation Systems, Automatic Incident Detection

1. INTRODUCTION

Visual surveillance for traffic systems requires short processing time, low processing cost and high reliability [1]. Under those requirements, image processing technologies offer a variety of systems and methods for Intelligence Transportation Systems (ITS) as a platform for traffic Automatic Incident Detection (AID). An extensive survey of the methods can be found in [2]. According to different traffic data sources, there exist two classes of AID methods mainly studied: one is based on inductive loops, radars, infrared sonar and microwave detectors [3] and the other is based on video images [4]. The first class of methods suffers from drawbacks in that they are expensive to install and maintain and they are unable to detect slow or stationary vehicles. Video sensors, on the other hand, offer a relatively low installation cost with little traffic disruption during maintenance. Furthermore, they provide wide area monitoring allowing analysis of traffic flows and turning movements, speed measurement, multiple-point vehicle counts, vehicle classification and highway state assessment, based on precise scene motion analysis.

The vast majority of traffic videos analysis methods segment the motion field into objects' trajectories. Hence, scene's motion reasoning is achieved by aggregating the specific objects' motion information [5,6,7,8]. The different trajectories are then analyzed to detect slow motion, swift changes in speed or trajectories interference for generating alerts.

Examination of each individual object's motion in the scene is a complicated task, which might be not practicable under real-time constrains. When the acquired scene is a complex enough, as, for example, highway interchange or junctions, those problems become a real obstacle on the way to real-time implementations. Some of the methods and real life

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applications try to overcome this problem by limiting the examination area in the image to a single line or small neighborhood defined by the user [6]. The shortcoming of those solutions is that events outside those areas will not be detected. Additionally those methods are fundamentally limited by the accuracy that can be achieved by the motion estimation method used. The latter problem can be dealt with by regarding the entire motion field rather than the different vehicles' trajectories. A recent work has suggested analyzing the entire motion field by wavelet decomposition and not by motion segmentation [9]. However the rigid structure of the wavelet basis functions might not be appropriate for the task in hand.

Traffic models, which describe the relationship among traffic stream characteristics, such as flow, speed and concentration, are the foundations of traffic research for the last 40 and 50 years [10]. Those models consist of mathematical micro- and macroscopic description of road conditions. While a consolidate traffic theory is available, none of the image processing techniques, presented for traffic analysis problems, has suggested utilizing traffic flow models into traffic video systems.

This paper presents a robust method for AID system, exploiting classical traffic flow models. Traffic flow models draw the relationships among the traffic stream variables, and characterize the road and its condition. The road conditions, as they are manifested in the model's parameters, are used for detection of incidents and abnormal behavior. Sect. 2 introduces traffic theory and Sect. 3 describes how traffic parameters are revealed in data acquired by video-based transportation systems, based on the image processing methods, while Sect. 4 presents a 3D traffic model which is utilized for estimation of road conditions and for automatic incident detection (AID) applications.

2. CHARACTERIZATION OF TRAFFIC STREAMS IN VIDEO

Traffic models describe the relationship among traffic stream characteristics. In discussing the models, the link between theory and measurement capability is important since often theory depends on measurement capability. A general notion of these variables, based on the intuitive idea self-evident from their names, will suffice for the purpose of discussing their measurement. Precise definitions of the variables of interest, which are used within the scope of the research, are given in Sect. 3. Five measurement procedures are discussed in this section:

- measurement at a point;
- measurement over a short distance (less than 10 meters);
- measurement over a length of road (more than 0.5 kilometer (km));
- the use of an observer moving in the traffic stream;
- wide-area samples obtained simultaneously from a number of vehicles.

The types of measurement are illustrated in a space-time diagram in Figure 1. The vertical axis of this diagram represents distance from some arbitrary starting point along the road, in the direction of travel. The horizontal axis represents elapsed time from some arbitrary star moment. Each line within the graph represents the trajectory of an individual vehicle, as it moves down the road over time. The slope of the line is that vehicle's velocity. Where lines cross, a faster vehicle has overtaken and passed a slower one. (The two vehicles do not in fact occupy the same point at the same time.) Measurement at a point is represented by a horizontal line across the vehicular trajectories: the location is constant, but time varies. In its earliest applications, video cameras were used to acquire the data in the field, which was then subsequently played back in a lab for analysis. In these early implementations, as illustrated in Figure 2, lines were drawn on the video monitor screen (literally, when manual data reduction was used). More recently this has been automated, which nowadays allows the data reduction to be conducted simultaneously with the data acquisition [6].

Measurement over a short section is represented by two parallel horizontal lines a very short distance apart. With video camera technology, two detector 'lines' placed close together provide the same capability for measuring speeds.

A vertical line represents measurement along a length of road, at one instant of time, such as a single snapshot taken from above the road. Measurements along a length of road come either from aerial photography, or from cameras mounted on tall buildings or poles. On the basis of a single frame from such sources, only density can be measured. Once several frames are available, speeds can also be measured.

The moving observer technique is represented by one of the vehicle trajectories, the heavy line in Figure 1. The wide-area samples are similar to having a number of moving observers at various points and times within the system.

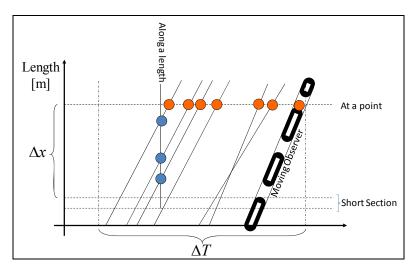


Figure 1 – Methods for obtaining traffic data



Figure 2 – Traffic measurement at a point in video sequences.

3. VARIABLES OF INTERESTS

In general, traffic streams are not uniform, but vary over both space and time. Because of that, measurement of the variables of interest for traffic flow theory is in fact the sampling of a random variable. In reality, the traffic characteristics that are labeled as flow, speed, and concentration are parameters of statistical distributions.

3.1 Flow Rates

Flow rates are collected directly through point measurements, and by definition require measurement over time. They cannot be estimated from a single snapshot of a length of road. Flow rate, q, is the number of vehicles counted, divided by the elapsed time, T:

$$q = \frac{N}{T} \tag{1}$$

Flow rates are usually expressed in terms of vehicles per hour, although the actual measurement interval can be much less. Figure 3(a) illustrates a time-space diagram, where the vertical and horizontal axes are the temporal and spatial axes respectively. The diagram depicts, for each time value (frame), the gray-level values of the pixels along the virtual line, Ω , which is presented, in black, in Figure 2. The number of the cars which passed, through the line throughout the

measuring period, T, can be measured by evaluating the temporal cross sections of the diagram. Such a cross-section is presented in Figure 4(a). When evaluating the gray-levels crossings, one can conclude that it might be difficult to extract the number of passing cars from the time-space diagram of the original sequence.

Real-time video scene segmentation to moving objects and static areas was presented in [11]. Segmentation is achieved through a statistical analysis of the motion vectors' magnitudes and angles, which results in formation of magnitude and angle motion segmentation driven masks. The combination of those masks, thus formulating the Real-Motion-Separation-Mask (RMSM), gives a tool that robustly extracts real-moving objects in real-time. The RMSM of Figure 2 is depicted in

Figure 5, where the line, Ω , is drawn in black and real moving objects are represented in darker pixels. The corresponding RMSM's time-space diagram and its cross-section are presented in Figure 3(b) and Figure 4(b), respectively. While the car counting task is difficult when considering the gray-level values' time-space diagram, (illustrated in Figure 3(a)), when considering the RMSM's diagram (Figure 3(b)), this task becomes significantly easier. Therefore, it is evident that the RMSM, developed for stabilization and SR purposes, is a strong tool for flow measurement.

In order to proceed further and simplify the flow computation even more, Athol's assumption [12] of uniform vehicle length is relaxed to a uniform width. Under this assumption, the flow is given by:

$$q = \frac{\sum_{x,y \in \Omega, t \in T} \left[\left(RMSM(x,y,t) - RMSM(x,y,t-1) \right) > Thr \right]}{\alpha T}$$
(2)

where [(RMSM(x,y,t)-RMSM(x,y,t-1))>Thr] is the number of pixels, with change in motion larger than a predefined threshold, along the virtual line, Ω , in time period T and α is the average car's width. Without loss of generality α is set to be 1. Additionally, since frames are acquired in fixed time periods, the elapsed time, T, is measured by the number of frames. By virtue of these assumptions, the flow can be computed from a video sequence by:

$$q = \frac{\sum_{x,y \in \Omega, t \in T} \left[\left(RMSM(x,y,t) - RMSM(x,y,t-1) \right) > Thr \right]}{N_{frames}}$$
(3)

Figure 6 illustrates the computation of the flow-rate in two real-life traffic scenarios. Figure 6 (a) and (b) are frames extracted from real-life video feed of a highway in free-flow and congestion conditions ([13]). Figures (c) and (d) are the corresponding RMSM frames, where real motion corresponds to brighter pixels. The video frame rate is 5 Hz, hence 300 frames per minute. The flow-rates for 300 frames, which Figures (a) and (b) are two samples of, are shown in figures (e) and (f). The average flow rates for each set are presented in Table 1. It is quite evident that the flow-rates for the situation described in figure (b) are higher.

3.2 Speeds

Measurement of the speed of an individual vehicle requires observation over both time and space. The instantaneous speed of an individual vehicle is defined as:

$$u_{i} = \frac{dx}{dt} = \lim_{(t_{2} - t_{1}) \to 0} \left(\frac{x(t_{2}) - x(t_{1})}{t_{2} - t_{1}} \right)$$
(4)

In the literature, the distinction has frequently been made between different ways of calculating the average speed of a set of vehicles. The kind of difference that can arise from different methods can be illustrated by the following example. If a traveler goes from A to B, a distance of 20 km, at an average speed of 80 kilometers per hour (km/h), and returns at an average speed of 40 km/h, what is the average speed for the round trip? The answer is of course not 60 km/h; that is the speed that would be found by someone standing at the roadside with a radar gun, catching this car on both directions of the journey, and averaging the two observations. The trip, however, took 1/4 of an hour one way, and 1/2 an hour for the return, for a total of 3/4 of an hour to go 40 km, resulting in an average speed of 53.3 km/h.

The first way of calculating speeds, namely taking the arithmetic mean of the observation,

$$\overline{u}_t = \frac{1}{N} \sum_{i=1}^N u_i \tag{5}$$

is termed the *time mean speed*, because it is an average observations taken over time. The second term that is used in the literature is *space mean speed* and is given by the harmonic mean of the individual vehicle speeds, as follows:

$$\overline{u}_{s} = \frac{D}{\frac{1}{N} \sum_{i} t_{i}} = \frac{D}{\frac{1}{N} \sum_{i} \frac{D}{u_{i}}} = \frac{1}{\frac{1}{N} \sum_{i} \frac{1}{u_{i}}}$$
(6)

The definition of space mean speed involves taking the average of the speeds of all of the vehicles on a section of road at one instant of time [14]. In Figure 1, this method is represented by the vertical line labeled "along a length". In deriving this however, an "isoveloxic" model is assumed, one in which each car follows a linear trajectory in the space time diagram, and is not forced to change speed when overtaking another vehicle. This is equivalent to assuming that the speed distribution does not change with location. A similar definition of space mean speed, without the isoveloxic, is the arithmetic mean of the speeds of vehicles occupying a given length of lane at a given instant [15].

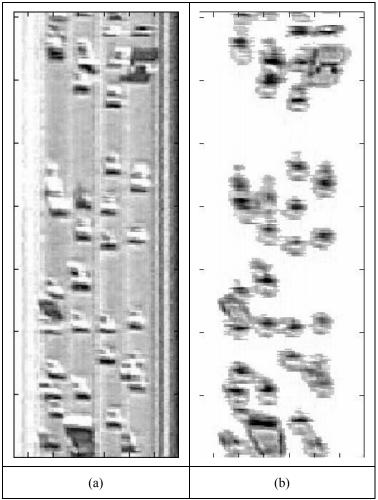


Figure 3– Time – Flow measurement through space-time diagram.

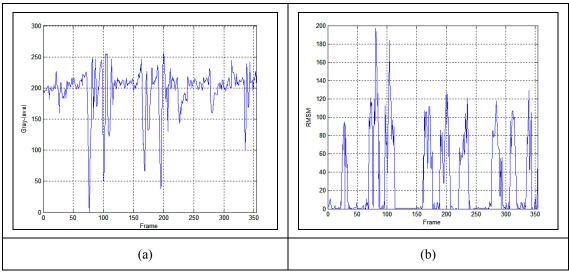


Figure 4 – Temporal cross section of the time-space diagram

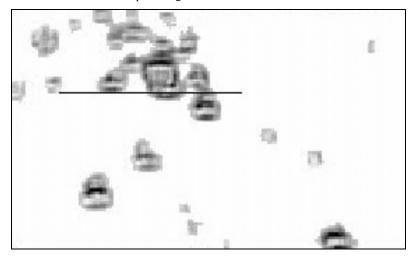


Figure 5 – Real Motion Separation Mask (RMSM) of the frame presented in Figure 2. Darker pixels represents pixels where motion was detected.

Regardless of the particular definition put forward for mean speed, the individual speeds are computed by the optical flow methods [16,17,18,19,20,21]. The computation of dense optical flow is very time consuming and might not comply with real-time constrains. To this end, following the discussion above, two methods for extracting the average speed can be utilized. The first method exploits the harmonic mean of speeds measured at a point over time and computes the optical flow only in the vicinity of the virtual line. The second method averages the speeds of all of the vehicles on a section of road at one instant of time, hence one frame. To avoid the situation, where the computation of the optical flow of the entire frame exceeds the system's capability, the two-stage real-motion extraction mechanism, described in [11], is utilized and the optical flow is computed only for pixels which are suspected to contain real motion.

Figure 7 illustrates the speed rates for 300 frames over two different minutes in which Figure 6 (a) and (b) were taken in. Evaluating Figure 7(a), one can conclude that the traffic conditions described in Figure 6 (a) allow specific drivers to exceed their speed, while the fluctuations in the graph presented in Figure 7 (b) suggest that the situation described in Figure 6 (b) corresponds to "stop and start" [22] conditions, which means that traffic is stopped and released in a periodic manner with varying periods and duty cycles.

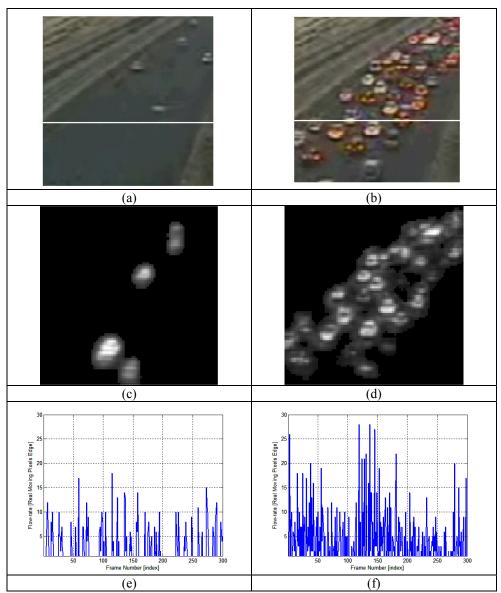


Figure 6 – Ayalon highway vision-based flow-rate computation. Figures (a) and (b) are two frames extracted from real-life video feed of Ayalon highway in free-flow and congestion conditions. Figures (c) and (d) are the corresponding RMSM frames. The flow-rates computed over 300 frames of the two different minutes, in which figures (a) and (b) where taken, are shown in figure (e) and (f) respectively.

Table 1 - Flow rate computed using a video stream for the two traffic scenarios described in Figure 6 (a) and (b)

Traffic Condition	Average Flow-rate
Figure 6 (a)	2.2458
Figure 6 (b)	4.3355

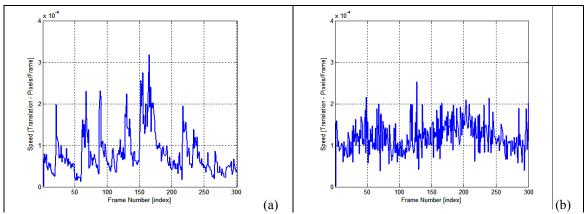


Figure 7 – Vision-based Speed rates computed over two different minutes, in which Figure 6(a) and (b) were taken in.

3.3 Concentration

Concentration has in the past been used as a synonym for density [23]. It seems more useful to use 'concentration' as a broader term encompassing both density and occupancy. The first is a measure of concentration over space; the second measures concentration over time of the same vehicle stream. Density can be *measured* only along a length. If only point measurements are available, density needs to be calculated from speed and flow [23]:

$$k = \frac{q}{\overline{u}_s} \tag{7}$$

The difficulty with using this equation for density estimation is that the equation is exactly correct only under some very restricted conditions, or in the limit as both the space and time measurement intervals approach zero. If neither of those situations holds, then the use of the equation to calculate density can give misleading results, which would not agree with empirical measurements. It follows that q equals $u \cdot k$ for the continuous surface, at a point. Real traffic flows, however, are not only made up of finite number of vehicles surrounded by real spaces, but are inherently stochastic [24]. Measured values are averages taken from samples, and are therefore themselves random variables. Measured flows are taken over an interval of time, at a particular place. Measured densities are taken over space at a particular time. Only for stationary processes (in the statistical sense) will the time and space intervals be able to represent conditions at the same point in the time-space plane. Hence it is likely that any measurements that are taken of flow and density (and space mean speed) will not be very good estimates of the expected values that would be defined at the point of interest in the time space plane – and therefore that Eq. (7) will not be consistent with the measured data.

The use of video camera technology allows measuring the actual concentration over a road length in a given time. This is achieved by dividing the number of pixels containing real motion in a given frame by the total number of pixels of the road. Without loss of generality, it is assumed that the road occupies the entire frame. This simplifies the image processing task and eliminates the need to segment the road in the video images, hence concentration is measured by dividing the number of pixels in the real motion separation mask (RMSM), which are equal to 1 by the image size.

Figure 8 illustrates the vision-based concentration computation over the 300 frames taken over two different minutes, in which Figure 6(a) and Figure 6(b) were taken. The minute average rates are presented in Table 2. Evidently the concentration rates in Figure 6(b) are higher. Similarly to the notion derived from the speed graphs, the concentration graph also implies that the situation described in Figure 6(b) can classified as "stop and start" [22], hence traffic is held and released in a periodic manner with random duty cycles.

4. SPEED – FLOW – CONCENTRATION TRAFFIC MODEL

Since the seminal work of Greenshields [25] a significant amount of work has been invested in effort to establish the relationship between the variables described in the previous section. Some of these efforts begin with mathematical models; others are primarily empirical, with little or no attempt to generalizing.

Generally speaking, the current status of mathematical models for speed-flow-concentration relationships is in a state of flux. The models that dominated the discourse for nearly 30 years are incompatible with the data currently being obtained, and with currently accepted depictions of speed-flow curves, but no replacement models have yet been

developed. Measuring the three parameters, flow, speed and concentration simultaneously, allows indicating the road current conditions over a 3D, flow-speed-concentration (FSC), space. Recognition of three-dimensional relationships is important for improved understanding. Consequently, it is important to make more use of those sets of freeway data in which all three variables have been measured and no estimation is needed. The simple image processing methods, suggested above, for traffic video analysis offer the ability to do that.

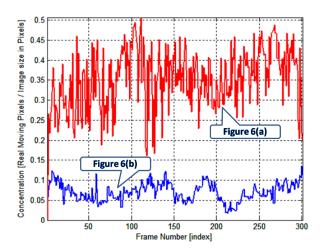


Figure 8 - Vision-based Concentration computed over 300 frames taken in the two different minutes in which figures Figure 6(a) and (b) were taken in.

Table 2 - Concentration rates computed using a video stream for the two traffic scenarios described in Figure 6 (a) and (b)

Traffic Condition	Concentration
Figure 6 (a)	0.0702
Figure 6 (b)	0.3521

The graph depicted in Figure 9 presents 12 points which correspond to the average flow, speed and concentration (FSC) rates computed over a video feed of 12 minutes, where both free-flow and "stop and start" traffic are present. The applicability of the method is illustrated in Figure 10 where 3 frames extracted from the video sequence are presented. Figure 10 (a), (b) and (c) are of the frames taken in the corresponding minute, where the FSC parameters for points (A), (B) and (C), shown in Figure 9, were computed over. One can see the different traffic conditions presented in Figure 10: free-flow (a), reasonably free flow with higher concentration rate (b) and congestion (c) are represented in three distinguished points on Figure 9.

5. TRAFFIC APPLICATIONS

5.1 Automatic Incident Detection

Figure 11 is a frame extracted from the sequence, one of which earlier frames is shown in Figure 2. As one can see, an accident is taking place on the right hand side of the image. The effect of the accident on traffic is obvious. For each frame of the sequence the flow, speed and concentration are measured by digital image processing means. Flow is computed at the line marked on Figure 11. Then, each frame is placed on the FSC-space according to its measured traffic parameters. This is depicted in Figure 12. There are two sets of markings collections. The first collection, marked with solid line represent frames taken before the accident took place, while the second one, marked with dashed line, is of frames taken after the accident occurred. Evaluating Figure 12, one can segment the frames projections on the FSC-space. As traffic incidents impact tends to persist for several minutes, automatic incident detection (AID) alarm can be triggered after a certain readings of the parameters are far from the normal, free-flow average, marked with the big circle on the figure.

6. CONCLUSION

In this paper, the basic concepts of traffic theory and their utilization in video based systems are outlined. First, the variables are introduced by which traffic flow is described. Sequentially, the paper describes how those variables can be measured from traffic video streams. Having the traffic variables measured based on the flow-speed-concentration 3D model, a robust AID scheme is suggested as well as measuring the traffic class of service. The results show that integration of available traffic flow models into the decision mechanism of video based intelligent transportation systems shows a great potential for both infrastructure traffic systems.

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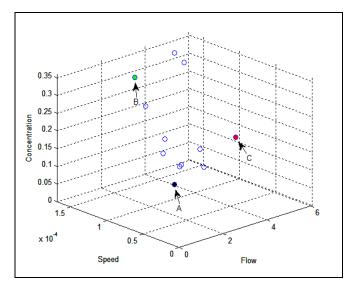


Figure 9 - Flow-Speed-Concentration (FSC) 3D model. Each point of the points on the graph represents the flow, speed and concentration average values computed over a minute is a 12 minutes real-life video feed of Ayalon highway, which contains free-flow as well as "stop and start" traffic. Speed in given as [Distance in Pixels/Frame], Flow as [Total number of Pixels in the RMSM edge /Frame] and concentration is the average of the ratio (number of Real-moving pixels)/(Frame Size in Pixels).

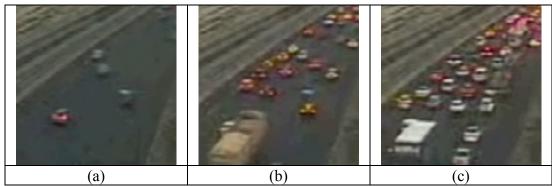


Figure 10 - The road conditions for the points (A), (B) and (C) in Figure 9.



Figure 11 - Traffic incident scenario.

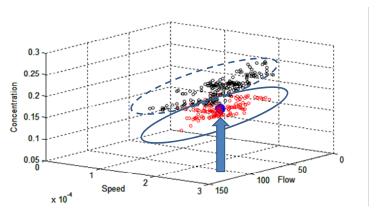


Figure 12 - Flow-Speed-Concentration 3D space of a video sequence capturing an accident. The traffic parameters are measured for every frame in the sequence. The lower batch of markings are of frames taken before the accident took place, while the black ones are of frames taken after the accident occurred. The blue circle represents the normal situation, free-flow, average of the three traffic parameters. Speed is given as [Distance in Pixels/Frame], Flow as [Total number of Pixels in the RMSM edge /Frame] and concentration is the average of the ratio (number of Realmoving pixels)/(Frame Size in Pixels).

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